THE VISIBILITY OF STARS WITHOUT OPTICAL AID

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INTRODUCTION

It is generally stated that the faintest stars readily visible to the unaided eye under ordinary conditions are of the sixth visual magnitude. To judge from the limiting magnitudes of the majority of the well-known naked-eye catalogs as well as from common experience, this statement is, indeed, substantially correct. The faintest stars in Ptolemy's *Almagest,* for example, are of magnitude 5.4; those in al Sûfi's catalog are of magnitude 5.6. The more modern catalogs have somewhat fainter limits. The faintest stars in Argelander's *Uranometria Nova* are, on the average, of magnitude 5.7, and those in the *Atlas Coelistis Novus* of Heis (who was noted for his keen eyesight) are of magnitude 6.1. Houzeau, in his *Uranométrie Générale,* has recorded stars of magnitude 6.4, and Gould, who many times remarked about the exceptionally clear atmosphere at Córdoba, states that persons of ordinarily good vision could see stars of the seventh magnitude. He has recorded in his *Uranometria Argentina* stars of magnitude 7.2. K. Lundmark[1] has remarked that he can see M33, which has a total magnitude of 6.8, and P. Meesters,[2] from a comparison of the Harvard Variable Star Charts with the sky, finds that he can see white stars of magnitude 6.8 and yellow stars of magnitude 6.9.

While the faintest stars visible to these latter observers are considerably below the commonly stated sixth magnitude limit, there are examples of even fainter astronomical objects than these being seen without optical aid.

H. D. Curtis,[3] in his experiments on the limit of unaided vision, attached to the tube of the 12-inch telescope of the Lick

[1] *Handbuch der Astrophysik,* **V/1**, 354, 1932.

[2] *De in Nederland met het bloote oog zichtbare sterren.* Amsterdam (1921); quoted in *Handbuch der Astrophysik,* **V/1,** 354, 1932.

[3] *Lick Obs. Bull.,* **2**, 67, 1901.

Observatory two fairly large blackened screens. The front screen, 178 inches distant from the rear one, contained at its center a hole one-fourth inch in diameter; the rear screen contained at its center a hole one-half inch in diameter. These apertures were aligned so that when a star was seen centrally placed in them, it was also found at the intersection of the cross wires of the three-inch finder. A movement of the telescope amounting to two or three minutes of arc was sufficient to carry a star out of the field of view formed by the apertures.

To use the apertures the observer clamped the telescope at the declination of the star to be examined. With his eye at the rear aperture he then moved the telescope slowly in hour angle and tried to pick up the star. When the star was seen (or thought to be seen) through the apertures, its position was noted in the finder. If it appeared not more than one or two minutes of arc from the intersection of the cross wires, the observation was considered successful.

Without using the diaphragms and apertures, Curtis was able to see stars of magnitude 6.5 directly against the background of the sky. With their aid he easily observed stars in the magnitude range 7.2 to 7.4. Four stars between magnitudes 7.9 and 8.1 were usually seen with little difficulty, two of magnitude 8.3 were "seen with difficulty," and one of magnitude 8.9 was "glimpsed at intervals, very doubtful." From these observations it would appear that, if seen upon a perfectly black background, a star of magnitude approximately 8.5 would be at the limit of unusually good vision.

H. N. Russell[4] has also determined the brightness of the faintest star visible to the unaided eye, but by a method very different from that used by Curtis. Russell placed a disc of white cardboard on black cloth and determined the maximum distance from the eyes at which it could be seen when illuminated by light from a star of known magnitude admitted through a small aperture into a darkened room.[5] From the data obtained

[4] *Pop. Astr.*, **10**, 242, 1902; *Ap. J.*, **45**, 60, 1917.

[5] It will be noted that Russell's experiment apparently differs slightly from that of Curtis. Russell, as far as can be determined from his description, used binocular vision; Curtis used monocular vision.

from the experiment (the size of the disk, its reflectance, the measured distance at which it just became invisible, and the magnitude of the star serving as the source of illumination) it was possible to determine the magnitude of the faintest star visible to the unaided eye. From several observations Russell found, in excellent agreement with Curtis, that a star of magnitude 8.5 seen against a black background is at the limit of visibility.

MODERN LABORATORY DATA ON VISUAL THRESHOLDS

During the past few years a wealth of information[6] on the visibility of objects of a great variety of sizes and contrasts and under various intensities of background illumination has been gathered under the auspices of the armed services and the Office of Scientific Research and Development. These purely laboratory data, combined with measurements of the brightness of the sky, may be used for a re-examination of the entire question of the limit of visibility of stars.

It has been found that an object of stellar size, in order to be just visible against a background of brightness b (measured in millimicrolamberts, mμL), must produce at the eye an illumination i (measured in foot candles) related to b by the equations:[7]

$$\log i = -9.80 + 2 \log (1 + 0.1122\, b^{\frac{1}{2}}), \quad \log b \leqq 3.17;$$

$$\log i = -8.35 + 2 \log (1 + 0.001122\, b^{\frac{1}{2}}), \quad \log b \geqq 3.17.$$

Two equations are necessary to represent the variation of i with b because of the nature of the vision process. At b equal to approximately 1500 mμL there is a rather abrupt change from foveal to extra-foveal vision, that is, from day vision to night vision. This change profoundly affects the observer's ability to see faint objects.

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[6] See, for example: H. A. Knoll, R. Tousey, and E. O. Hulburt, "Visual Thresholds of Steady Point Sources of Light in Fields of Brightness from Dark to Daylight," *J.O.S.A.*, **36**, 480, 1946; also, H. Richard Blackwell, "Contrast Thresholds of the Human Eye," *J.O.S.A.*, **36**, 624, 1946.

[7] These formulae are the analytical expressions of Knoll, Tousey, and Hulburt's data as given by Selig Hecht (*J.O.S.A.*, **37**, 59, 1947).

With the value of a metercandle in terms of stellar magnitude derived some years ago by Russell,[8] the relationships between i and b given above may be converted into relationships between stellar magnitude, m, and sky brightness, b. The variation of the magnitude of a star at the limit of visibility as a function of sky brightness is given numerically in Table I, and is illustrated graphically in Figure 1. Through use of these data it is possible, given the brightness of the sky background, to predict the magnitude of the star at the limit of vision for the average observer.

TABLE I

THE LIMITING MAGNITUDE, m, VISIBLE TO AN AVERAGE OBSERVER USING UNAIDED VISION, AS A FUNCTION OF SKY BRIGHTNESS, b, IN MILLIMICROLAMBERTS

$\log b$	m
$-\infty$	7.7
0	7.5
1	7.0
2	6.1
3	4.3
3.17	4.1
4	3.9
5	3.4
6	2.5
7	0.8
8	−1.3
9	−3.7
10	−6.2

The limiting magnitude, 7.7, found in Table I for a completely black sky ($\log b = -\infty$) agrees satisfactorily with the value 8.5 found by Curtis and by Russell. The discrepancy of 0.8 magnitude is readily attributable to individual differences of observers. Both Curtis and Russell, to judge from the data available, had very good night vision. The spread in the individual values of m (for $b = 0$), from which the mean value 7.7 given in Table I is derived, is from magnitude 8.3 to 7.5. A similar spread has been found by other observers. Langmuir

[8] *Ap. J.*, **41**, 103, 1916. One metercandle is equivalent to a star of magnitude −14.18. For comparison, the magnitude of the full moon is −12.6.

and Westendorp[9] have, in a similar case, observed a range of 0.6 magnitude, with values of individual observers lying between the limits 8.0 and 7.4.

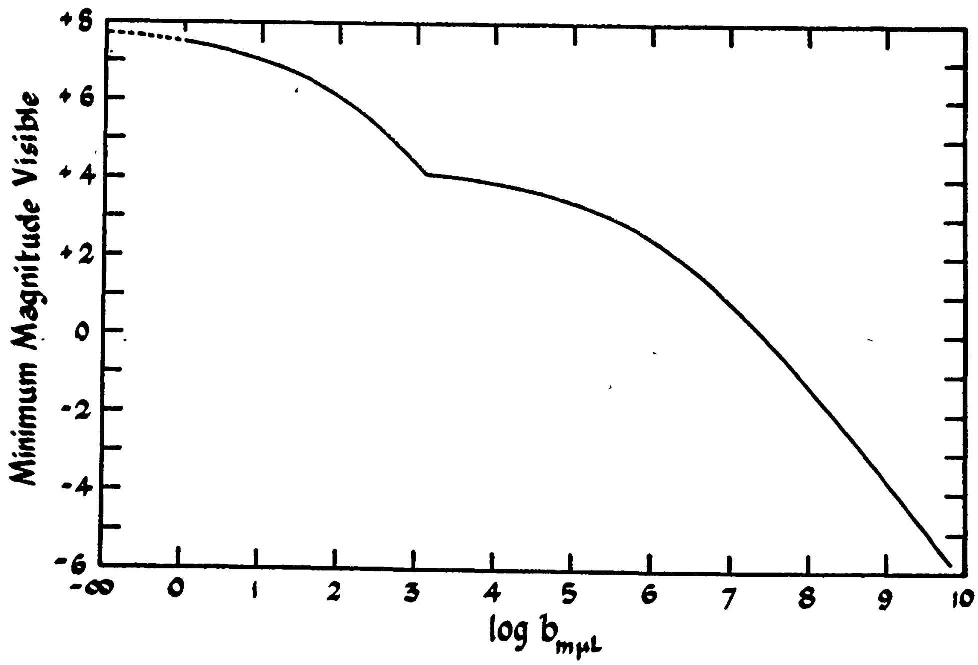

FIG. 1.—Minimum magnitude visible for different background brightness, b, measured in millimicrolamberts, mμL.

THE VISIBILITY OF STARS AT NIGHT

The brightness of the night sky, upon a knowledge of which depends our use of Table I, and to which the limit of vision is quite sensitive, exhibits a considerable variation with time of night, season, and year. It likewise varies with the intensity of terrestrial magnetic activity, auroral activity, and haziness of the atmosphere. It is critically dependent upon the phase and position of the moon, and it varies at all times with the zenith distance and azimuth. A series of visual measurements[10] of the brightness of the night sky (Table II) from the University of

[9] *Physics,* 1, *273,* 1931.

[10] Although a fairly extensive literature on the brightness of the night sky exists, most of the more recent measurements are of little value in this study, for they were made with photocells having a wave-length sensitivity very different from that of the eye.

Rochester[11] will, however, serve as a guide to the brightness of the night sky under three selected conditions.

TABLE II

SELECTED VALUES OF THE VISUAL BRIGHTNESS OF THE NIGHT SKY*

Condition	Observed Range log b		Average log b
Clear moonless	2.00	2.16	2.08
Clear moonlight, age 10^d, moon near horizon	2.48	3.00	2.74
Clear moonlight, age 10^d, moon near meridian† ...	2.84	3.56	3.20

* The values of b in mμL refer to a region of the sky at zenith distance $\zeta = 45°$.
† The altitude at which this places the moon is not known.

These data, referred to the zenith[12] and combined with those of Figure 1, yield the limiting magnitudes, visible under the specified conditions, in Table III.

TABLE III

MAGNITUDES OF STARS AT THE LIMIT OF VISION UNDER SPECIFIED CONDITIONS

Conditions (All Observations Made at the Zenith)	Limiting Magnitude for Limits of log b in Table II		Limiting Magnitudes for Average log b
Clear moonless	6.2 mag.	6.0 mag.	6.1 mag.
Clear moonlight, age 10^d, moon near horizon....	5.5	4.6	5.2
Clear moonlight, age 10^d, moon near meridian...	4.9	4.0	4.2

The average value determined from these laboratory data, 6.1 mag. for a clear moonless night, agrees remarkably with the value 6.0 mag. generally given for such conditions.[13]

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[11] "A Series of Measurements of the Night Sky"; report compiled under Contract OEM-rd 265 with the National Defense Research Committee.

[12] The corrections applied are the averages of those determined from the observations of J. Rudnick (*Ap. J.* **87**, 584, 1938) and of J. Dufay (*Bull. Obs. d'Lyon,* **10**, 1928), and from the theory of the daylight sky developed by R. Tousey and E. O. Hulburt (*J.O.S.A.,* **37**, 78, 1947).

[13] The brightness of the sky increases from the zenith to the horizon. This increase, combined with the extinction of the starlight caused by the

The magnitudes quoted in Table III illustrate clearly the dependence of the limits of vision upon sky conditions, particularly upon the position and age of the moon. The decrease in the total number of stars visible in the three cases listed in Table III is very impressive. On a clear moonlit night with a ten-day moon near the horizon (limiting magnitude 5.2) only 38 percent as many stars are visible as on a clear moonless night (limiting magnitude 6.1); on a clear moonlit night with a ten-day moon near the meridian (limiting magnitude 4.2) only 13 percent as many stars are visible as on a clear moonless night.[14]

The zodiacal light and the Milky Way also may have a considerable effect on the limit of visibility. The intensity of the zodiacal light varies over a considerable range. On the average, it is perhaps five to ten times as great as that of the night sky.[15] Thus in the brighter parts of the zodiacal region the limit of visibility is reduced by a magnitude or more, and the general influence of the zodiacal light in reducing the limit of visibility (by the processes mentioned in note 13) may amount to several tenths of a magnitude.

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increased airpath the light must traverse as one observes nearer the horizon, means that the limit of visibility away from the zenith will not be quite so faint as magnitude 6.1.

The fact that the sky brightens as the horizon is approached will have still another effect on the limiting magnitude visible. The magnitude values quoted in Table III were computed as though the observer were examining a screen evenly illuminated and of the intensity stated in the table. Strictly speaking, this is not the case; for the observer, in looking at the zenith, for example, sees parts of the sky that are brighter than the zenith. This means that the eye is not normally adapted to the brightness of the zenith, but to some slightly higher brightness. The effect of this nonuniform sky illumination is thus to change the actual limiting magnitude at the zenith to a value slightly brighter than the one tabulated.

[14] The data on numbers of stars brighter than a given magnitude used in these calculations are from the observations of Seares, van Rhijn, Joyner, and Richmond, *Ap. J.*, **62**, 320, 1925.

[15] This and other estimates of the brightness of the zodiacal light and the Milky Way have been taken from papers by: E. O. Hulburt, *Phys. Rev.*, **35**, 1098, 1930; C. T. Elvey and F. E. Roach, *Ap. J.*, **85**, 213, 1937; C. T. Elvey, *Ap. J.*, **86**, 84, 1937; and C. T. Elvey and Paul Rudnick, *Ap. J.*, **86**, 562, 1937.

The limit of visibility may also be reduced by several tenths of a magnitude in the region of the Milky Way. The effect of the light of the Milky Way on estimates of stellar magnitudes is well illustrated in visual photometric catalogs, which very generally show a systematic magnitude error of one or two tenths of a magnitude in the vicinity of the Milky Way. This error is directly traceable to the brightening of the general sky background as the Milky Way is approached.

THE VISIBILITY OF STARS IN DAYLIGHT

The data of Table I go to values of b sufficiently high to allow us to make some calculations of the visibility of stars in daylight. Here again, however, as for the night sky, it is difficult to take into account the large and unpredictable variations in brightness for any given region of the daylight sky caused primarily by the variable amount of haze in the atmosphere. Only average or, preferably, theoretical values of the daylight sky brightness can be quoted.

In Table IV are given values of the brightness of the daylight zenith sky as it would appear from an altitude of 10,000 feet and at sea level, and for various zenith distances, ζ, of the sun. The

TABLE IV

THE BRIGHTNESS OF THE DAYLIGHT ZENITH SKY FOR "CLEAR" AIR, AND FOR VARIOUS ZENITH DISTANCES, ζ, OF THE SUN

ζ	Altitude 10,000 Feet log b	Sea Level log b
0°	8.73	8.89
30°	8.67	8.83
60°	8.50	8.65
75°	8.36	8.51

tabulated values have been computed from the theoretical expressions for the brightness of the sky developed by Tousey and Hulburt.[16] These equations have been found to give results for the brightness of the sky in excellent agreement with observations.

[16] *J.O.S.A.*, 37, 78, 1947.

The values tabulated do not refer to "pure" air, which was never found by Tousey and Hulburt during any of their measurements of sky brightness, but for what Tousey and Hulburt have called "clear" air, which has an attenuation coefficient about 35 percent greater than that of "pure" air. "Clear" air was, in fact, to judge from the published data, found only at altitudes of 2000 feet or more; the air at lower altitudes was always hazy, and the sky brightness consequently greater than the value predicted for "clear" air. It is possible, of course, that at some localities "clear" air may exist even at sea level.

In Table V the limiting magnitudes corresponding to the values of b in Table IV have been tabulated. In addition, the limiting magnitudes visible from sea level through a "slightly hazy" atmosphere are given. A "slightly hazy" atmosphere has been one assumed to give a zenith sky brightness twice that of a "clear" atmosphere. Under ordinary conditions this "slightly hazy" atmosphere would probably appear to the average observer to be the normal one; the "clear" atmosphere would probably appear to be very transparent. To judge from the observations of Tousey and Hulburt,[16] the "slightly hazy" atmosphere is the one generally found to exist at sea level, at least in the region in which their observations were made.

TABLE V

THE LIMITING STELLAR MAGNITUDE VISIBLE IN THE ZENITH DURING DAYLIGHT

ζ	Altitude 10,000 Feet	Sea Level	Sea Level, Slightly Hazy
0°	—3.0 mag.	—3.4 mag.	—4.2 mag.
30°	—2.8	—3.2	—4.0
60°	—2.5	—2.8	—3.6
75°	—2.1	—2.5	—3.2

The limiting magnitudes given in Table V make it clear that there is no hope of seeing any *star* without optical aid, except a bright nova during daylight hours (at least with $\zeta \leq 75°$). Several of the planets, however, are, according to the data of Table V, visible during the daytime under the proper circumstances.

Venus, ranging in brightness[17] from magnitude —3.3 to —4.3, should, if located in the zenith, be visible at any time through "clear" air, and, when brightest, even through "slightly hazy" air. Mars, of magnitude —1.1 to —2.8 at time of opposition, should be visible during a part of the day. Likewise Jupiter, varying between magnitudes —1.4 to —2.5 at time of opposition, should be visible during a small fraction of the day.

Table V is limited in its usefulness in predicting the daylight visibility of astronomical objects since it refers only to the zenith. It is difficult to prepare a table of sky brightness for zones of the sky at varying zenith distances and for various altitudes of the sun, inasmuch as the sky brightness at any given zenith distance varies considerably with azimuth.[18] It is possible, however, to construct a table of *average sky brightness* for a given zenith distance, ξ, of the area of the sky under observation, and zenith distance, ζ, of the sun. This average sky brightness can be found by determining the average brightness of regions of constant zenith distance spaced 45 degrees apart in azimuth. In Table VI will be found average sky brightness values com-

TABLE VI

AVERAGE BRIGHTNESS OF THE DAYLIGHT SKY AS SEEN FROM AN ALTITUDE OF 10,000 FEET, FOR VARIOUS ZENITH DISTANCES, ξ, OF THE AREA UNDER OBSERVATION, AND FOR VARIOUS ZENITH DISTANCES, ζ, OF THE SUN

log b

ξ	$\zeta=0°$	$\zeta=30°$	$\zeta=60°$
0°	8.73	8.67	8.50
30°	8.74	8.69	8.56
60°	8.85	8.84	8.80
80°	9.17	9.19	9.20

[17] The data on the brightness of the planets have been taken from Russell, Dugan, and Stewart, *Astronomy*, **1** (Ginn and Company, 1945).

[18] As an example of the variation to be expected, the sky brightness at altitude = 10,000 feet, zenith distance of area under examination = 40°, zenith distance of sun = 60°, is, as a function of azimuth, Z (measured from the direction to the sun):

Z	0°	45°	90°	135°	180°
log b	8.76	8.70	8.58	8.53	8.54

puted in this manner; in Table VII will be found the corresponding limiting magnitude values taken from Figure 1, and corrected for extinction.

TABLE VII

LIMITING MAGNITUDES (EXTINCTION INCLUDED) CORRESPONDING TO BRIGHTNESS—VALUES OF TABLE VI

ξ	$\zeta=0°$	$\zeta=30°$	$\zeta=60°$
0°	−3.0 mag.	−2.8 mag.	−2.5 mag.
30°	−3.0	−2.9	−2.6
60°	−3.4	−3.3	−3.2
80°	−4.7	−4.8	−4.8

THE VISIBILITY OF STARS IN TWILIGHT

A series of observations of the brightness of the sky at the zenith and on the horizon after sunset, made recently by E. O. Hulburt[19] of the Naval Research Laboratory, may be used to determine the rate of appearance of stars with the fading of twilight. The results of combining Hulburt's sky-brightness data with those of Figure 1 are illustrated graphically in Figure 2, in which the magnitude of a star at the limit of visibility[20] in the zenith is plotted as a function of zenith distance of the sun. The values of the limiting magnitude for a zenith star during daylight hours have been taken from Table V, and they have been corrected to apply to an altitude consistent with that at which Hulburt's observations were made.

Two features of this curve are noteworthy: (1) The change from foveal to extra-foveal vision is clearly shown on the curve in the region of zenith distance 102°. The apparent effect of this

[19] These observations were made by Dr. Hulburt near Bocaiuva, Brazil (altitude about 2200 feet), during the time the National Geographic–U.S. Army Air Corps 1947 Eclipse Expedition was located there, and were reported at the conference on the upper atmosphere held in London July 7–10, 1947. It is a pleasure to thank Dr. Hulburt for a copy of his London paper in advance of publication.

[20] The faintest star visible in the zenith has been placed at magnitude 6.0 in agreement with the particular sky-brightness measurements used in the construction of the figure rather than the average value $6^{m}.1$ quoted in Table III.

change in vision process is not large, however, because of the rapid decrease in zenith sky brightness as the sun goes to still greater zenith distances. (2) The zenith sky attains its night level when the sun is 18° below the horizon, the point at which astronomical twilight is said to end.

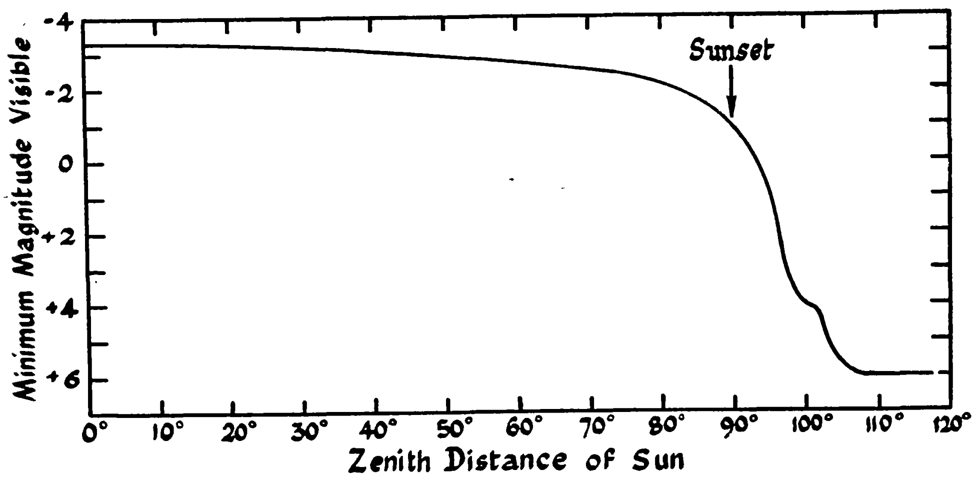

FIG. 2.—Variation of minimum magnitude visible in the zenith with zenith distance of the sun.

The faintest stars visible on the western horizon after sunset when the zenith distance of the sun was greater than 108° were about 0.8 mag. brighter than the faintest stars visible in the zenith. At the eastern and northern horizons, however, the faintest stars visible when the zenith had reached its night level were only 0.6 mag. fainter than the faintest visible zenith stars. This difference of 0.2 mag. between the western and the northern-eastern horizons was caused by the zodiacal light and the Milky Way, both of which were visible when Hulburt's observations were made; the effect demonstrates further the marked influence of these light sources upon the visibility of faint objects.